The magnitude of this relative increase in absorbance at high frequencies in the normal state is stronger in our films than in the single-crystal measurements of Joyce and Richards. In our case the energy to the bolometer changes by 12% in the gap region and by 20% at 210 cm^{-1}. The relative strength of the peaks at 55 and 87 cm^{-1} changes with the age of the film (stored at 77 °K). This effect is related to an increase of reflectivity at low frequency relative to high frequency and is more marked in the superconducting state. As a result, after several days the 89 cm^{-1} peak appears to weaken.

Joyce and Richards propose a mechanism for the structure where the radiation generates electronic excitations of energy 2Δ plus a phonon of energy $\hbar\Omega$. They found however a disagreement of some 6 cm^{-1} between the $\hbar\Omega+2\Delta$ expected and their observed structure. We find no such disagreement in our measurements. From tunneling data[2] the main peaks in $\alpha^2F(\Omega)$, the quantity proposed to describe the process, occur at 56 and 87.5 cm^{-1} in excellent agreement with our peaks at 55 and 87 cm^{-1}. The additional structure we observe can be correlated with various phonon features seen in tunneling and neutron scattering but our signal-to-noise ratio (1000) does not permit an unambiguous identification of this structure. The discontinuity at 42 cm^{-1}, however, does not correspond to any known phonon singularity in lead but agrees with tunneling data. Our experimental method is very similar to that of previous investigators,[3] but the phonon structure described here is at somewhat higher frequency than the region of interest to those workers, and requires a high signal-to-noise ratio for identification.

In conclusion it appears that the phonon generation process found by Joyce and Richards is clearly observable in lead films and provides an alternative to tunneling measurements.

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Semiclassical Analysis of Field Emission through Atoms Adsorbed on Metal Surfaces*

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INTRODUCTION

Recent experiments on the field emission of electrons through atoms adsorbed on metal surfaces[1-3] have demonstrated the importance of a resonance tunneling mechanism.[1-5]

One simple theoretical approach employs a semiclassical (WKBJ) analysis of the one-dimensional potential shown in Fig. 1.[3-5] This is a problem that has been investigated by semiclassical techniques, not only in solid-state and surface physics,[3-7] but also in chemical physics when the potential energy barrier for a reactive collision contains a well,[8,9] and in nuclear physics when the fission barrier for heavy nuclei possesses two maxima.[10,11]

The purpose of this note is first to draw attention to this body of work on resonance tunneling in other branches of physics, and second to consider certain interesting features of resonance tunneling within the semiclassical approximation. In particular, for energies lying well below the barrier maxima, the transmission coefficient is expressed explicitly in terms of the resonance energies and widths of the quasistationary states that exist within the well of the potential [see Eq. (5) below]. This is in accord with the results of Refs. 2–5. The calculations of Ref. 9 are used.

TRANSMISSION COEFFICIENT

An expression for the transmission coefficient D valid for energies E lying above or below the barrier maxima $V(b_1)$ and $V(b_2)$ is considered first, and then specialized to the case when the energy

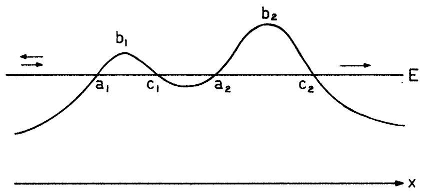

FIG. 1. One-dimensional potential $V(x)$. E is the energy of the system. Classical turning points are denoted by a_1, c_1, a_2, and c_2, while b_1 and b_2 are the positions of the barrier maxima. Arrows indicate incident, reflected, and transmitted waves.

lies well below the barrier maxima.

In Ref. 9, the following first-order semiclassical expression for the transmission coefficient was derived[12]:

$$D(E)=\frac{(T_1^2-1)(T_2^2-1)}{T_1^2T_2^2+1+2T_1T_2\cos[2\alpha-(\varphi_1+\varphi_2)]} , \quad (1)$$

where α is the phase integral across the well,

$$\alpha=\int_{c_1}^{a_2}k(x)\,dx \quad (2)$$

with

$$k(x)=\{(2m/\hbar^2)[E-V(x)]\}^{1/2} .$$

Also, we have

$$\begin{aligned} T_j&=(1+e^{2\pi\epsilon_j})^{1/2} , & j&=1,2 \\ \varphi_j&=\epsilon_j+\arg\Gamma(\tfrac{1}{2}+i\epsilon_j)-\epsilon_j\ln|\epsilon_j| , & j&=1,2 \end{aligned} \quad (3)$$

and

$$\left.\begin{aligned} &-\pi\epsilon_j=\int_{a_j}^{c_j}|k(x)|\,dx>0 \quad \text{when } E<V(b_j) \\ \text{or} \\ &-\pi\epsilon_j=\mathrm{Re}\,i\int_{ia_j}^{ic_j}k(x)\,dx<0 \quad \text{when } E>V(b_j) . \end{aligned}\right\} j=1,2$$

The subscript j refers to the first or second barrier, respectively. When the energy is greater than the barrier maxima, the classical turning points c_1 and a_2 in Eq. (2) are to be replaced by b_1 or b_2.[9]

Equation (1) is valid for energies greater or less than the barrier maxima, and hence could be used in a uniform theory of thermionic and field emission for the potential in Fig. 1, similar to that for single barrier passage.[13]

The function φ_j cancels a singularity in the phase integral α at $E=V(b_j)$. This singularity has important consequences in the closely related problem of orbiting collisions[14,15] and hence is known as the "orbiting singularity" and φ_j is called a "quantum correction function." Equations (3) are based on a parabolic approximation to a barrier maximum,[9,16] although other model forms could also be used.[17]

The scattering by the potential may be characterized in terms of the complex energy

$$\mathcal{E}=E_n-i\tfrac{1}{2}\Gamma_n , \quad E_n>0 , \quad \Gamma_n>0 ,$$

where E_n is the resonance energy and Γ_n is the level width, by the application of complex boundary conditions.[9,18] This operation leads to the following Bohr-Sommerfeld quantization condition for the complex eigenvalues:

$$\alpha=(n+\tfrac{1}{2})\pi+\tfrac{1}{2}(\varphi_1+\varphi_2)-i\,\tfrac{1}{2}\ln T_1T_2 , \qquad n=0,1,\ldots . \quad (4)$$

Equation (4) holds for energies above and below the barrier maxima. In the latter case when $\Gamma_n \ll E_n$, Eq. (4) simplifies to[9]

$$\int_{c_1}^{a_2}\left(\frac{2m}{\hbar^2}[E_n-V(x)]\right)^{1/2}dx=(n+\tfrac{1}{2})\pi+\tfrac{1}{2}(\varphi_{1n}+\varphi_{2n}) ,$$

a Bohr-Sommerfeld quantization condition for the well, and

$$\Gamma_n=\frac{\hbar\omega_n}{\pi}\ln T_{1n}T_{2n} ,$$

where ω_n is the classical angular frequency of oscillation in the well and the subscript n on ω, φ_j, and T_j indicates that $E=E_n$ in the definition of these quantities. In terms of E_n and Γ_n, the transmission coefficient through a resonance becomes[9]

$$D(E)=\frac{1}{X_n}\frac{(\tfrac{1}{2}\Gamma_n)^2}{(\tfrac{1}{2}\Gamma_n)^2+(E_n-E)^2} , \quad (5)$$

where $X_n \geq 1$ is an asymmetry factor defined by

$$X_n=(T_{1n}T_{2n}-1)^2/(T_{1n}^2-1)(T_{2n}^2-1) .$$

Equation (5) is of a Breit-Wigner (Lorentzian) form and clearly illustrates the resonance behavior of the transmission coefficient. The above equations illustrate the important features of resonance tunneling transmission in a straightforward way and may be regarded as a complement to the point-match calculations of Refs. 3 and 4.

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Influence of Combined Static and Time-Dependent Quadrupole Interactions on Angular Correlation in Nuclei with Integral Spin*

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The application of the Bloch-Wangsness-Redfield theory of nuclear relaxation to the study of perturbed angular correlations in even-A nuclei subject to simultaneous static and time-dependent quadrupole interactions has been treated. The differences in the perturbation factors as compared to the odd-A case are shown.

In a recent paper[1] (denoted below by I), we have treated the application of the Bloch-Wangsness-Redfield theory of nuclear relaxation to the study of perturbed angular correlations (PAC) in odd-A nuclei subject to simultaneous static and time-dependent quadrupole interactions. Stimulated by recent experimental results[2] we have investigated the extension of this formalism to the case of even-A nuclei (integral spins).

Problems may be expected in the case of integral spins due to the degeneracy of the hyperfine transitions $+\alpha \to 0$ and $-\alpha \to 0$, where α represents the spin projection m_j. The result of this degeneracy in NMR studies is that the resonance line corresponding to these transitions may have a shape made up of a superposition of Lorentzians,[3] which would correspond to the appearance of a combination of exponentials for the relaxation of the associated frequency in the perturbed angular-correlation spectrum. It is the purpose of this addendum to show that this effect will in fact occur in perturbed angular-correlation spectrum.

To demonstrate this, we review first the solution of the odd-A problem. The basic equation to be solved is (2.16) of I:

$$\dot{\rho}^*_{\alpha\alpha'} = \sum_{\beta\beta'} R_{\alpha\alpha'\beta\beta'}\,\rho^*_{\beta\beta'}\,, \quad \alpha - \alpha' = \beta - \beta' \tag{1}$$

where the matrix elements $R_{\alpha\alpha'\beta\beta'}$ are combinations of various spectral densities of the form

$$\mathcal{J}_{\alpha\alpha'\beta\beta'}(\omega) = \int_{-\infty}^{\infty} \langle(\alpha|K(t)|\alpha')(\beta'|K(t-\tau)|\beta)\rangle_e\, e^{-i\omega\tau}\, d\tau\,. \tag{2}$$